Multiple - i/p Multiple - o/p system

In continuous-time LTI system has m inputs p outputs, and N state variables, then a state space representation of the system can be expressed as.

$$\dot{q}(t) = Aq(t) + Bx(t)$$

$$y(t) = Cq(t) + Dx(t)$$

$$q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{bmatrix}, \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}_{N\times N}, \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ b_{N1} & b_{N2} & & b_{Nm} \end{bmatrix}_{N\times m}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \vdots & & & \\ c_{p1} & c_{p2} & \cdots & c_{pN} \end{bmatrix}_{p\times N}, \quad D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ d_{21} & d_{22} & \cdots & d_{2m} \\ d_{p1} & d_{p2} & \cdots & d_{pm} \end{bmatrix}_{p\times m}$$

$\Rightarrow$ The o/ps are $y_1(t) = 4q_2(t) + 3x_1(t)$

$y_2(t) = q_3(t) + 4x_2(t) + x_3(t)$

$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}}_{y(t)} = \underbrace{\begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}}_{q(t)} + \underbrace{\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 1 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}}_{x(t)}$$

$p \times m$

State variable - The smallest set of variable that determines the state of the system.

State vector - It is the vector which contains the state variable as element

Disad. ① complex computation
② many computation are reqd

Q. A MIMO system is described by the following differential eqns. Represent the system in state space.

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = x_1(t) + 3x_2(t) + 4x_3(t) \quad \text{— ①}$$

and o/ps are $y_1(t) = 4\frac{dy(t)}{dt} + 3x_1(t)$

$y_2(t) = \frac{d^2y(t)}{dt^2} + 4x_2(t)$

⇒ Select the state variable as, $q_1(t) = y(t)$

$\underset{= \dot{q}_1}{q_2(t)} = \dot{y}(t)$, $\underset{= \dot{q}_2}{q_3(t)} = \ddot{y}(t)$. $N = 3$

then $\dot{q}_1(t) = q_2(t)$, $\dot{q}_2(t) = q_3(t)$

$\dot{q}_3(t) = -4q_1(t) - 4q_2(t) - 3q_3(t) + x_1 + 3x_2 + 4x_3(t)$

$$\underbrace{\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \end{bmatrix}}_{\dot{q}(t)} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & -3 \end{bmatrix}}_{\substack{A \\ N\times N \\ 3\times 3}} \underbrace{\begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}}_{q(t)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 4 \end{bmatrix}}_{\substack{B \\ N\times m \\ 3\times 3}} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$x(t) \Rightarrow m$

$N = 3$
$m = 3$

$q(t) = \begin{bmatrix} q_1(t) \\ \vdots \\ q_N(t) \end{bmatrix}$,

⇒

A system is described by the differential eqn

$$\frac{d^3y(t)}{dt^3} + \frac{6d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 10y(t) = 8x(t)$$

where $y(t)$ is o/p and $x(t)$ as input. obtain the state space representation of system.

$N=3$, $N=3$, q_1, q_2, q_3

$q_1(t) = y(t)$, $q_2(t) = \dot{y}(t)$, $q_3 = \ddot{y}(t)$

state variable

Then $\dot{q}_1(t) = q_2(t)$, $\dot{q}_2(t) = q_3(t)$

$$\dot{q}_3(t) = -a_N q_1(t) - a_{N-1} q_2(t) - \cdots a_1 q_N(t) + x(t)$$

$$= -10q_1(t) - 11q_2(t) - 6q_3(t) + 8x(t)$$

The above 3X3 eqn can be written as

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \\ q_N(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \\ -a_N & -a_{N-1} & -a_{N-2} & -a_1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_N(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x(t)$$

3×1

system matrix (NxN), state vector (Nx1)

$$\dot{q}(t) \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -11 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}}_{q(t)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}}_{b} x(t)$$

$\dot{q}(t)$

$$y(t) = q_1(t) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C}^{1\times 3} \underbrace{\begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}}_{q(t)}^{3\times 1}$$

$$\left[\begin{aligned} \dot{q}(t) &= Aq(t) + b\,x(t) \\ y(t) &= Cq(t). \end{aligned}\right.$$